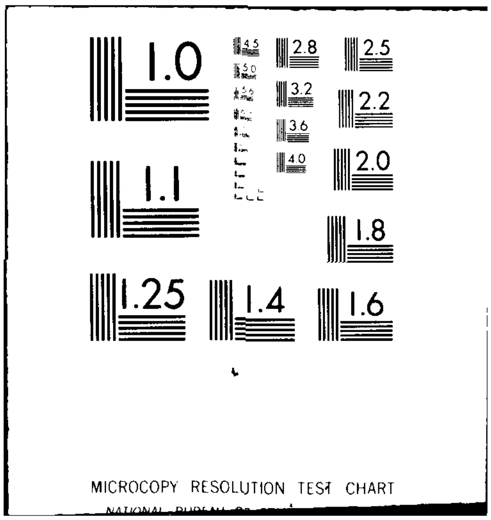
AD-A089 652   WISCONSIN UNIV-MADISON DEPT OF STATISTICS   F/G 12/1
A NOTE ON A DIFFICULTY INHERENT IN ESTIMATING RELIABILITY FROM --ETC(U)
AUG 80 B HARRIS, A P SOMS   N00014-79-C-0321
UNCLASSIFIED UWIS-DS-80-614   NL

1 OF 1
AD A
089652

END
DATE
FILMED
10-80
DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
MICROCOPY RESOLUTION TEST CHART

DEPARTMENT OF STATISTICS
UNIVERSITY OF WISCONSIN
Madison, Wisconsin 53706

TECHNICAL REPORT NO. 614
August 1980

A Note on a Difficulty Inherent in Estimating Reliability from Stress-Strength Relationships

by

Bernard Harris* and Andrew P. Soms**

*University of Wisconsin-Madison
**University of Wisconsin-Milwaukee

A Note on a Difficulty Inherent in Estimating Reliability from Stress-Strength Relationships

Bernard Harris and Andrew P. Soms

<u>Introduction</u>. Let X and Y be independent random variables with cumulative distribution functions $F_X(x)$ and $G_Y(y)$ respectively. The objective is to estimate

$$R = P\{Y < X\}. \tag{1}$$

This problem arises in the following physical context. Suppose that X is the strength of a component which is subjected to a stress Y. Then the component fails whenever $X \leq Y$ and does not fail when $Y < X$. For purposes of this exposition, it suffices to assume that X and Y are continuous random variables with probability density functions $f_X(x)$ and $g_Y(y)$.

It is easily established that

$$R = E_X\{G_Y(X)\} \tag{2}$$

and in many of the references given below, specific parametric models are employed, such as assuming that $F_X(x)$ and $G_Y(y)$ are both normally distributed. In this case

$$R = \Phi\left(\frac{\mu_X-\mu_Y}{\sqrt{\sigma^2+\sigma_Y^2}}\right). \tag{3}$$

The purpose of this note is to examine the consequences of such parametric assumptions.

Research supported by the Office of Naval Research under Contract No. N00014-79-C-0321 and the United States Army under Contract No. DAAG29-80-C-0041.

There is an extensive literature discussing both point and interval estimation of R. In particular, the reader is referred to G.K. Bhattacharyya and R.A. Johnson (1974), Z.W. Birnbaum (1956), Z.W. Birnbaum and R.C. McCarty (1958), S. Chandra (1975), J.D. Church and B. Harris (1970), F. Downton (1973), Z. Govindarajulu (1967), Z. Govindarajulu (1968), Z. Govindarajulu (1974), G.D. Kelley, J.A. Kelley and W.R. Schucany (1976), M. Mazumdar (1970), D.B. Owen, K.J. Craswell and D. Hanson (1964), H. Tong (1974). Numerical comparisons between several techniques are given in a survey paper by R.S. Downs and P.C. Cox (1974).

A number of examples have come to the attention of the authors, in which estimates using such parametric assumptions produced results which were significantly contradicted by subsequent experience. In this note, we exhibit a model, which illustrates how such departures from theory occur, yet still provide conformity with the experimental data. In some of these examples, the discrepancies have caused catastrophic results. For this reason, we feel that the publication of this note is warranted, despite its elementary nature.

2. <u>The Normal Stress-Strength Model</u>. To modivate the subsequent material, we present an example in which the normal model was employed. To simplify the discussion, without affecting any of the conclusions to be obtained, we will specify the parameters $\mu_X, \mu_Y, \sigma_X, \sigma_Y$. In practice these will naturally be

estimates and sampling errors will have to be accounted for in any inferential statement.

__Example__. Let $\mu_X = 84.6$, $\mu_y = 53.7$, $\sigma_X = 6.0$, $\sigma_y = 1.5$. Then from (3), $1-R = 3.036 \times 10^{-7}$, which naturally suggests a highly reliable component.

The normal model is justified on the basis of the central limit theorem. Clearly, the physical properties of stress and strength appear to satisfy the usual intuitive requirements for approximate normality. However, the mode of convergence implied by the central limit theorem is of the $\lim_{n\to\infty} F_n(u) = \Phi(u)$, where $\Phi(u)$ is the standard normal distribution and $u = (x-EX)/\sigma_X$. This does not preclude "large" relative errors in the tails. As indicated by the above example, the computation of the reliability R is determined almost entirely by the lower tail of $F_X(x)$ and the upper tail of $G_Y(y)$. Further, since *experimental* data obtained from random samples of strength and stresses (whether paired or two independent samples) is collected primarily from the "center" of the distribution, little experimental information about the tails tends to be available. As is exhibited in the next section, relatively small perturbations to the tail of the strength distribution can make the failure probability far higher than may be regarded as desirable, particularly in the case where failures can be catastrophic. We exhibit this phenomenon for the example given above.

3. __A Model For Stress-Strength Data Which Conforms to Many Practical Situations__.

For the equipment whose data is given in the example in Section 2, when it was placed into service, approximately one device in 1000 failed, contradicting

the results of the example in Section 2. A proposed explanation is given below.

Let $0 < \varepsilon < 1$ be given and let

$$F_X(x) = (1-\varepsilon)H(x) + \varepsilon K(x), \tag{4}$$

where $H(x)$ is a normal distribution function and

$$P(U < Y) > 1 - \eta, \tag{5}$$

$0 < \eta < 1$ and U is distributed by $K(x)$. The case of interest occurs when ε and η are both small. Physically, we may regard this as follows. With probability $1-\varepsilon$, the customary properties of the strength distribution hold, but with probability ε, a "flaw" is present and the device fails when subjected to a random stress distributed in accordance with $G_Y(y)$. Then, if a random sample of N_X strengths is observed, the probability that no device with a flaw is observed is

$$(1-\varepsilon)^{N_X} \sim e^{-N_X\varepsilon}. \tag{6}$$

and

$$(1-R) > \varepsilon(1-\eta) \sim \varepsilon. \tag{7}$$

To see how this behaves numerically, we return to the previous illustration and suppose $\varepsilon = .001$, $N_X = 100$. Then the probability that no flaw is detected is .905 and the failure probability of the device is .001 and not 3×10^{-7} as obtained earlier. Consequently, a naive application of inferential procedures based on (3) is highly likely to drastically overestimate the reliability (in the sense of a large relative error in the failure probability), since the presence of the alternate mode of failure (a flaw) is not likely to be detected during the experiment.

4. Concluding Remarks. This note is intended to exhibit a serious problem in the use of stress-strength relationships in the estimation of reliability. This problem is not arificial; it has actually occurred in a number of instances that have come to the attention of the authors. Further, it is clear that the usual types of experiment in which a random sample of N_X strengths and N_Y stresses is observed or N pairs of strength and stresses are observed will not eliminate the difficulty, unless astronomically large sample sizes are employed. Unfortunately, testing strength is frequently destructive of the item being tested so that increasing the sample size is economically infeasible.

In the example given in Section 2, the population parameters were assumed known. However, if sample estimates for $N_X = N_Y = 100$ are used, a 90% upper confidence limit for 1-R is easily seen to be about 7×10^{-7}. Thus the effect of not knowing the parameter changes the estimate from one failure in 3,000,000 to about one failure in 1,400,000. However, the introduction of the pertubation $\varepsilon K(x)$ changed the probability of failure to one on 1000. In addition, the difference between $F_X(x)$ and $H(x)$ will not be detected

by goodness-of-fit tests, thus giving the experimenter false confidence in the results given by the example of Section 2.

While the preceding discussion has treated the case of normally distributed stresses and strengths, the same kind of problem arises in other parametric models.

One way to proceed in circumventing this difficulty is to devise models for the existence of flaws, which can be tested statistically. Such studies are presently underway and some partial results have been obtained.

REFERENCES

Bhattacharyya, G.K. and Johnson, R.A. (1974). Estimation of reliability in a multicomponent stress-strength model. J. Amer. Stat. Assoc., 69, 966-970.

Birnbaum, Z.W. (1956). On a use of the Mann-Whitney Statistic. Proc. Third Berkeley Symposium on Mathematical Statistics and Probability, 1954-1955, Vol. 1, 13-17.

Birnbaum, Z. W. and McCarty, R.C. (1958). A distribution free upper confidence bound for P(Y<X) based on independent samples of X and Y. Ann. Math. Statist. 20, 558-562.

Chandra, S. and Owen, D.B., (1975). On estimating the reliability of a component subject to several different stresses (strengths). Naval Res. Logist. Quarterly, 22, 31-39.

Church, F.D. and Harris, B. (1970). The estimation of reliability from stress-strength relationships. Technometrics, 12, 49-54.

Downton, F. (1973). The estimation of P(Y<X) in the normal case, Technometrics, 15, 551-558.

Downs, R.S. and Cox, P.C. (1975). The probability of motor case rupture. Proc. Twentieth Conference on the Design of Experiments in Army Research Development and Testing, 801-824.

Govindarajulu, Z. (1967). Two sided confidence limits for P(Y<X) based on normal samples of X and Y. Sankhyā, Series B, 29, 35-40.

Govindarajulu, Z. (1968). Distribution free confidence bounds for P(X<Y). Ann. Inst. Stat. Math., 20, 229-238.

Govindarajulu, Z. (1974). Fixed width confidence intervals for P(X<Y), in Reliability and Biometry: statistical analysis of lifelength. Proc. Conf. Florida State Univ., Tallahassee, Fla. (1973), Soc. Ind. App. Math., 747-757.

Kelley, G.D., Kelley, J.A., and Schucany, W.R. (1976). Efficient estimation of P(Y<X) in the exponential case. Technometrics, 18, 359-366.

Mazumdar, M. (1970). Some estimates of reliability using interference theory. Naval Res. Logist. Quarterly 17, 159-165.

Owen, D.B., Craswell, K.J., and Hanson, D.L. (1964). Nonparametric upper confidence bounds for Pr(Y<X) and confidence limits for Pr(Y<X) when X and Y are normal. J. Amer. Statist. Assoc., 59, 906-924.

Tong, H. (1974). A note on the estimation of Pr(Y<X) in the exponential case. Techometrics, 16, 625.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: TR #614
2. GOVT ACCESSION NO.: AD-A089 652
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): A Note on a Difficulty Inherent in Estimating Reliability from Stress-Strength Relationships
5. TYPE OF REPORT & PERIOD COVERED: Scientific-Interim
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Bernard Harris and Andrew P. Soms
8. CONTRACT OR GRANT NUMBER(s): ONR N00014-79-C-0321; Army DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Statistics, University of Wisconsin, Madison, Wisconsin 53706
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, 800 N. Quincy Street, Arlington, VA 22217
12. REPORT DATE: August 1980
13. NUMBER OF PAGES: 8
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Distribution of this document is unlimited
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report):
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
Reliability
Stress-strength
Modelling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This note calls attention to a difficulty which arises frequently in the application of stress-strength methods in reliability theory. This difficulty has led to unanticipated catastrophic failures in a number of applications.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)